Generative Adversarial Networks as Priors for Inverse Problems

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Abstract

Direct observations of properties of porous media within the earth's interior are rare and therefore solving inverse problems is a common task in geoscience. Setting inverse problems in a Bayesian framework, the aim is to find the posterior distribution of rock properties given observed data.

This work aims to introduce a representation of the prior distribution given by a generative adversarial network (GAN). We show that GANs can be used to address many difficult problems in the geosciences, including seismic inversion (Mosser et al. 2018), generation of pore-space images (Mosser et al. 2017), and history matching in reservoir simulation.

GANs allow sampling from probability distributions that are implicitly defined by a set of example training images. In practice, GANs are represented by a pair of deep convolutional neural networks, a generator, and discriminator, which are trained in a competitive two-player setting.

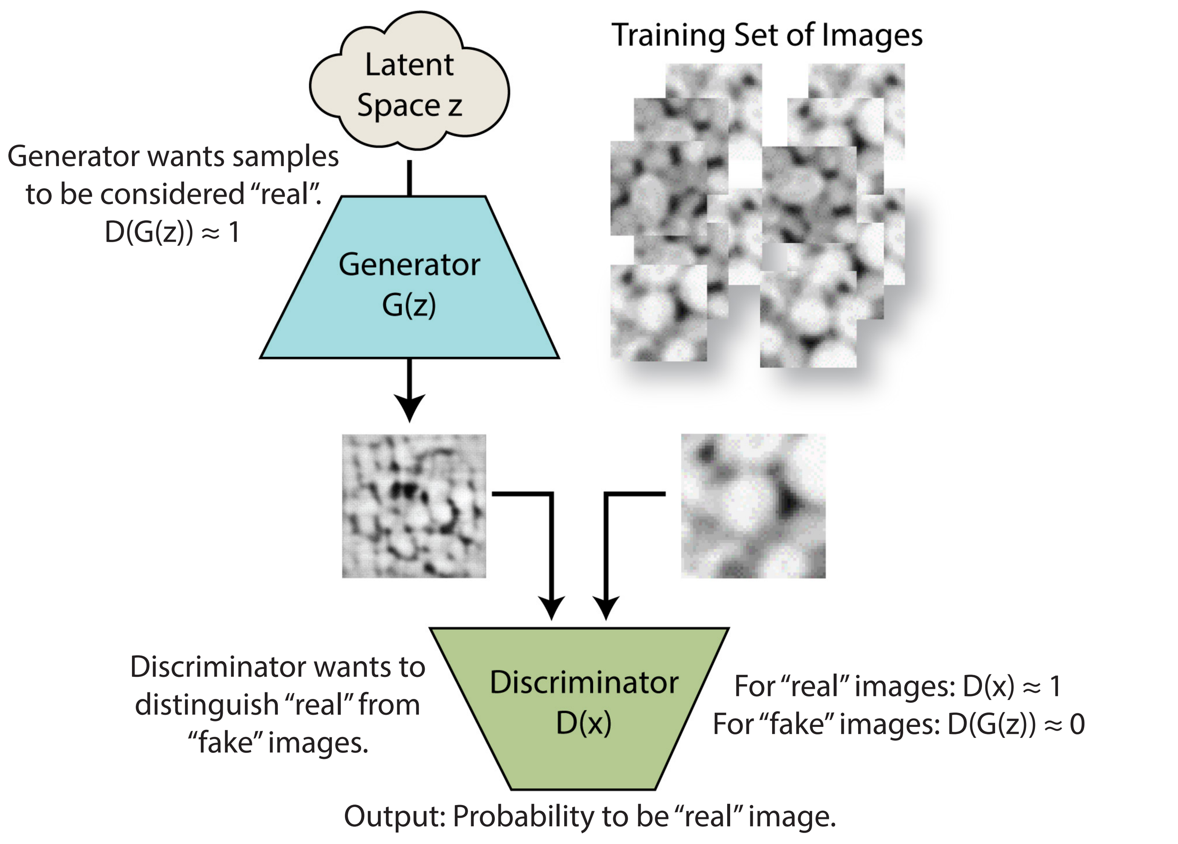
We show that a Metropolis-adjusted Langevin algorithm (MALA) allows stochastic solutions of the ill-posed seismic inversion problem at reservoir scale, constrained by the acoustic wave equation to be obtained. Our future work aims to extend this methodology to history matching of hydrocarbon reservoir production.

Generative Adversarial Networks (GAN)

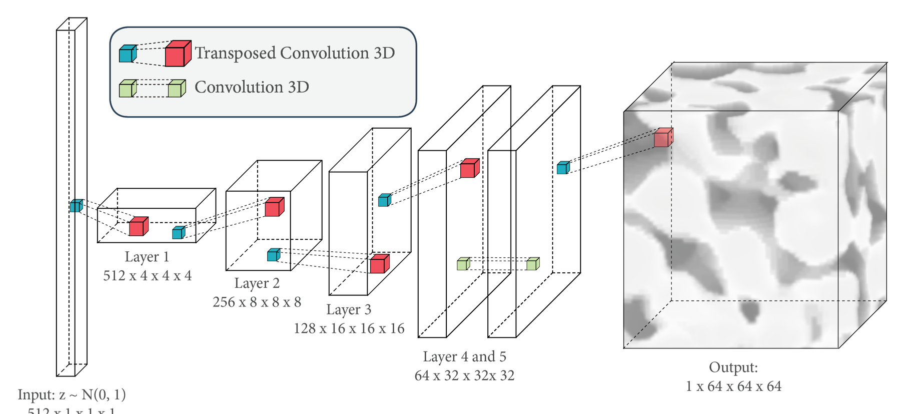
Generative adversarial networks provide a method of sampling from a probability distribution that is implicitly defined by a large set of training images. Two differentiable functions, a generator **G** and a discriminator **D**, play a competitive two-player minimax game.

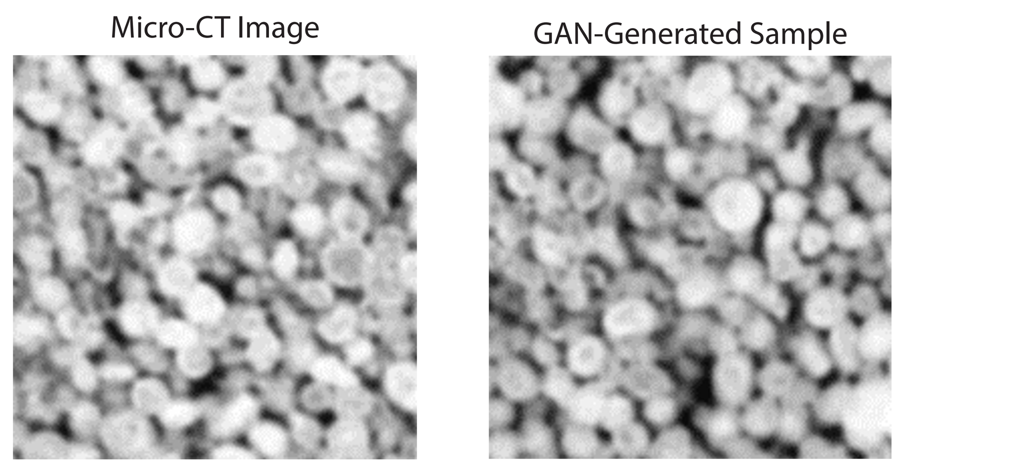
$$\min_{\theta}\max_{\omega}\{\mathbb{E}_{\mathbf{x}\sim p_{\mathbf{x}}}[\log\ D_{\omega}(\mathbf{x})]+\mathbb{E}_{\mathbf{z}\sim p_{\mathbf{z}}}[\log\ (1-D_{\omega}(G_{\theta}(\mathbf{z})))]\}$$

The generator creates samples by mapping samples drawn from a normal distributed latent space **z** to the space of natural images. The discriminator tries to maximize his abililty to distinguish samples from **G** and can be seen as a learned loss function.



We train GANs to create stochastic representations of porous media at the pore-scale. The generator and discriminator are represented by deep convolutional neural networks. The networks were trained on micro-CT images of a Ketton Limestone (64^3 voxels).

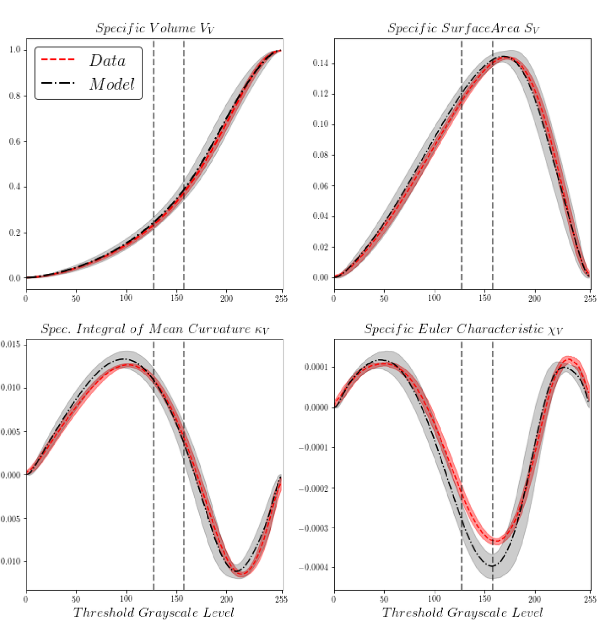




Above: The original micro-CT image used as a training set (left) and the GAN generated samples are visually nearly indistinguishable. Small scale features such as existing micro-porous regions are present in the GAN generated samples.

The resulting samples have high visual fidelity and show a diverse set of microstructural features of the porous medium seen in the training images.

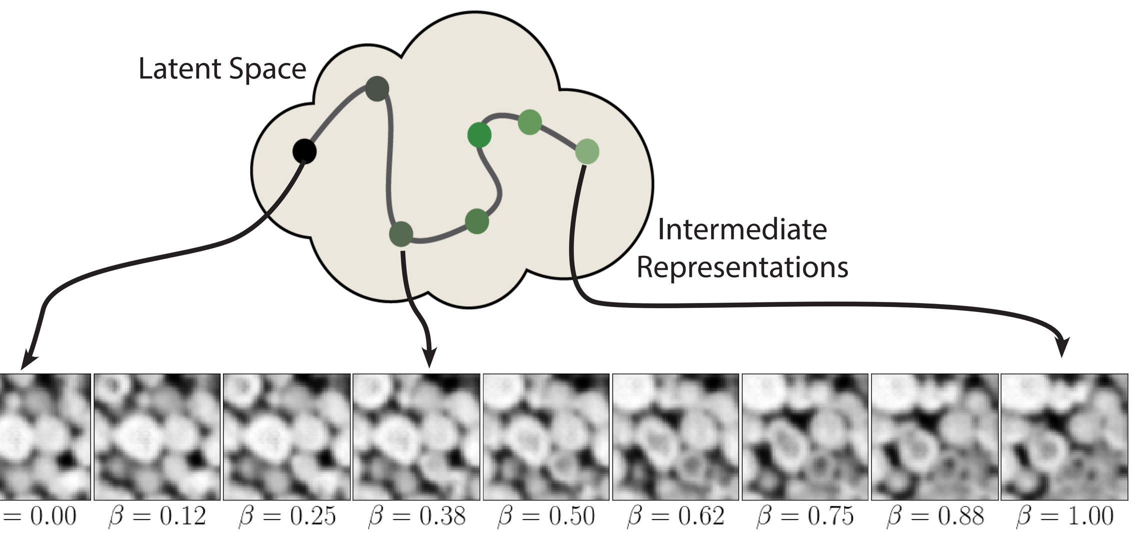
We evaluate statistical measures and Minkowski functionals and compute single phase permeabilities to ensure that the generated images reflect the original porous medium.



GitHub.com/LukasMosser/PorousMediaGAN

Conditioning Generative Models

The generator maps any point in the latent space **z** to the space of images. Interpolation between points in latent space results in interpolation in the image domain where each intermediate step is a sample of the implicit probability distribution defined by the training set.



Above: We show an example interpolation between two points in the latent space **z** for the GAN trained on the Ketton limestone. A number of intermediate steps along a linear path between the start and end points are shown. We observe a smooth interpolation in the space of generated porous media samples.

The differentiable nature of the deep neural network used to represent the generator allows the generator to be used for optimization problems with differentiable loss functions.

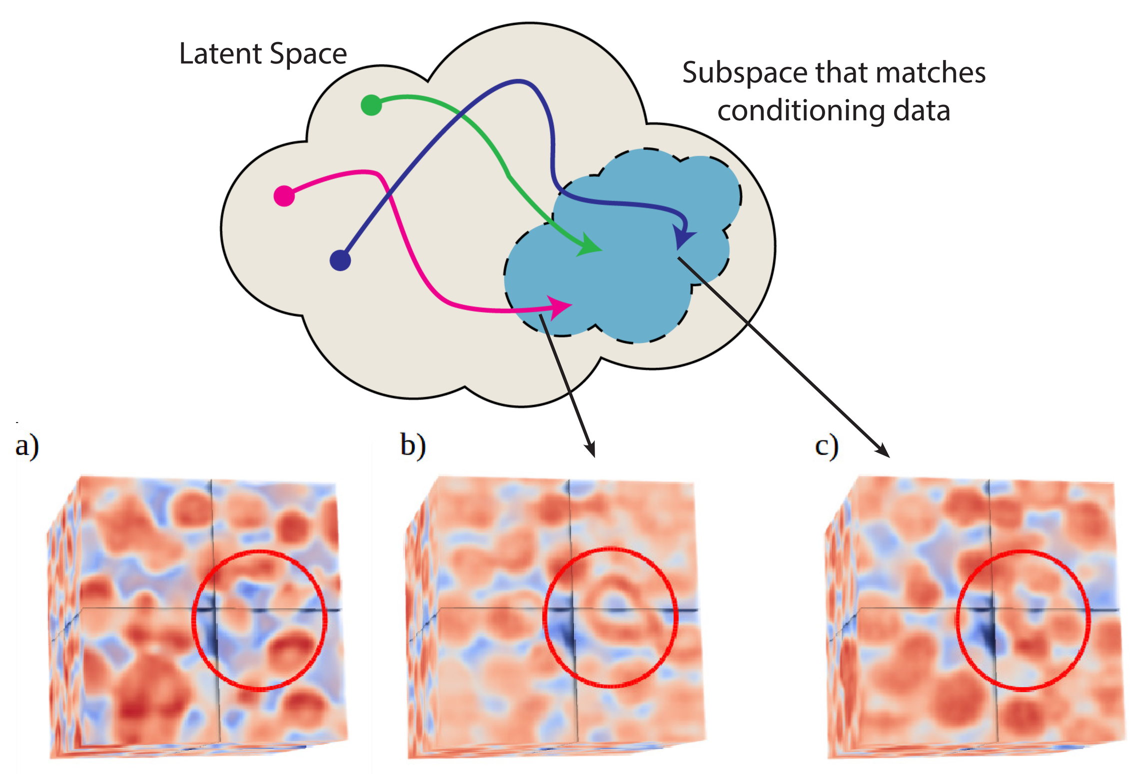
The generated GAN samples can be conditioned to existing data by minimizing the content loss, represented by the L_2- norm between the existing data and the GAN generated output. A mask **M** ensures that the contextual loss is only accounted for where spatial data is available.

$$L_{content}=\|G(\mathbf{z})\odot\mathbf{M}-data\|$$

Visual and statistical fidelity is ensured by a second objective function, the so-called perceptual loss, is given by the discriminator output. The resulting samples should receive a score **D(G(z))** ≈ 1.

$$L_{perceptual}=log(1-D(G(\mathbf{z})))$$

Optimization is performed by sampling and modifying a random latent vector **z** by computing gradients with respect to the contextual loss and the perceptual loss by backpropagating through the differentiable discriminator function.



Three orthogonal cross-sections were extracted from a Ketton limestone image (a) and used to condition a generative model. The contextual and perceptual losses were optimized by modifying the latent random variables. Starting from different initial random latent vectors leads to stochastic samples in the three-dimensional domain (b-c). Highlighted in red (b-c), two resulting features constrained to the same 2D conditioning data.

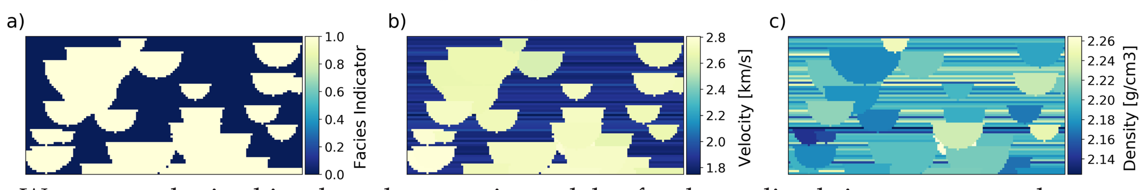
GitHub.com/LukasMosser/GeoGAN

Stochastic inversion - GANs + PDEs

Stochastic inversion seeks to obtain samples of the posterior distribution of rock properties e.g. the spatial distribution of rock p-wave velocity or permeability, given observed data, combined with a prior representing our belief of what possible distributions of the quantities of interest may look like. This is summarized by Baye's rule:

$$p(\mathbf{z}|\mathbf{d})=\frac{p(\mathbf{d}|\mathbf{z})p(\mathbf{z})}{p(\mathbf{d})}\propto p(\mathbf{d}|\mathbf{z})p(\mathbf{z})$$

To obtain samples from the posterior, we apply a Metropolis-adjusted Langevin algorithm (MALA). The prior is given by a GAN pre-trained on synthetic river channel systems and dependent only on the set of latent variables. The observed data is given by a forward simulation on an unseen geological cross-section (below).



We use synthetic object based reservoir models of a channelized river system to demonstrate stochastic PDE-based inversion. A GAN was trained to create stochastic samples of reservoir rock properties; reservoir facies (a), rock p-wave velocity (b) rock density (c).

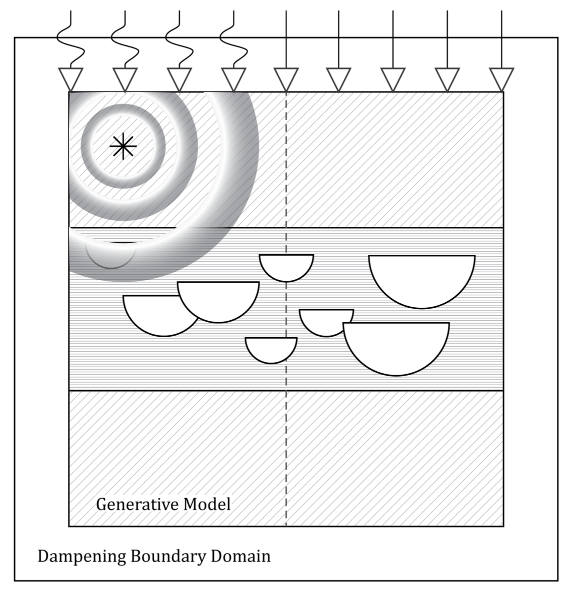
Each step update to perform MALA requires gradients of the mismatch with respect to the latent variables z to be obtained:

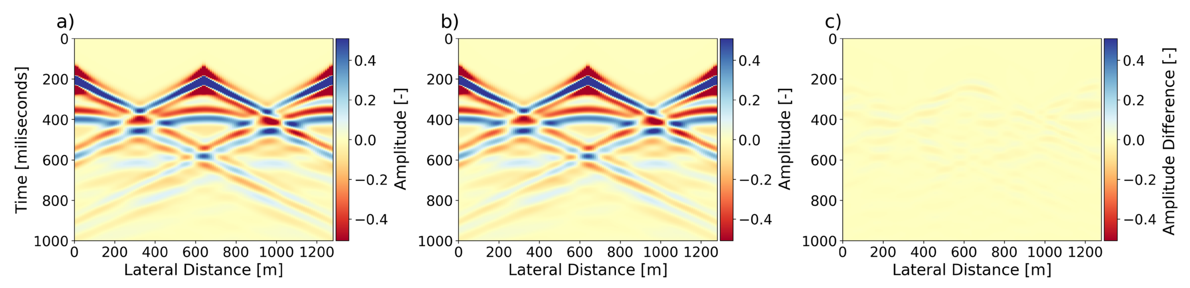
$$\mathbf{z}_{t+1}=(1-\lambda)\mathbf{z}_t+\epsilon_1\frac{\partial\|S(G_{\theta}(\mathbf{z}_t))-d^{obs}\|_2}{\partial G_{\theta}(\mathbf{z}_t)}\frac{\partial G_{\theta}(\mathbf{z}_t)}{\partial\mathbf{z}_t}+\mathcal{N}(0,\epsilon_2)$$

The deep neural network and forward PDE solver are coupled in a computational graph and gradients obtained by applying the adjoint state-method.

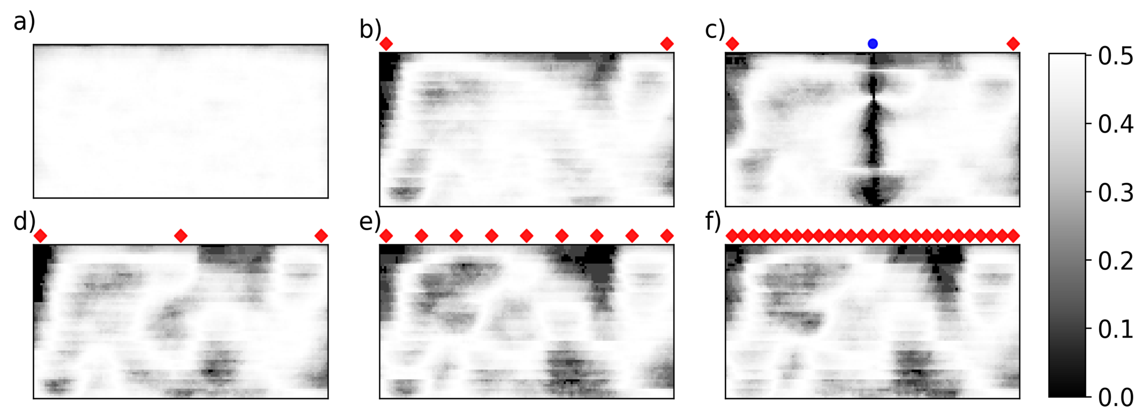
MALA - Posterior Sampling

We perform stochastic inversion using the MALA sampling approach for a reservoir scale acoustic wave propagation problem (right). The reflected wave-field is sparsely sampled by a number of recording devices at the surface. This represents an ill-posed inverse problem. The generative network acts as a prior on subsurface structures and allows samples from the posterior to be obtained that match the observed data.





Above: Comparison of ground truth recorded acoustic wave-form (a) obtained from solution of the acoustic wave-equation on a reservoir scale domain with inverted wave-form obtained from MALA-sampling (b). The obtained samples have < 10% relative error compared to the observed ground truth data (c).



Above: Standard deviation of 100 inverted reservoir models for an increasing number of acoustic sources (red diamonds). 100 unconditional samples obtained from the generative model show a high standard deviation. Increasing the number of sources leads to better resolved geological structures (b, d-f). Inverted samples can also be conditioned to well-data (blue circle) where available (c).

History Matching in Latent Space

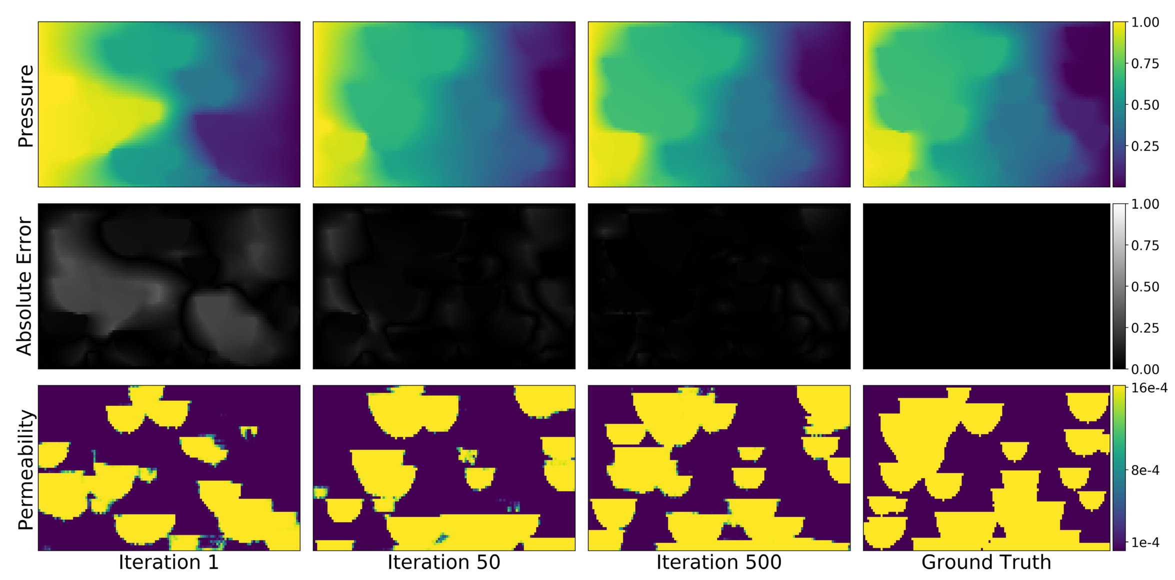
Our future work aims to use generative models as geological priors for reservoir-scale inverse problems such as history matching of hydrocarbon reservoir production data.

As a first step we solve a well-posed inverse problem of Darcy flow at the reservoir scale with Dirichlet boundary conditions:

$$-\nabla\cdot(\frac{\bar{k}(G(\mathbf{z}))}{\mu}\nabla p),\ \ in\ \Omega=[0,1]^2$$

$$p_D=1-x,\ \ for\ x\in\partial\Omega$$

The spatial distribution of permeability is sampled from a GAN generator trained on fluvial river channel systems. Using the adjoint-state equation, latent variables are modified to optimize the mismatch between the observed pressure field and the generated data by solving Darcy flow using a numerical finite-element solution on the GAN generated permeability field. The resulting channel systems closely match the ground truth permeability data.



Above: Inversion of reservoir permeability is performed for a 2D Darcy flow problem. The target ground truth permeability distribution on the right is reproduced by inversion using a GAN as a prior model on reservoir permeability. Inversion progress is shown from the initial starting model (left) until convergence is reached (right).

Conclusions

- The differentiable nature of generative adversarial networks and their latent vector representations allows challenging problems in the geosciences to be addressed such as seismic inversion or pore-space image generation.
- GANs represent a flexible methodology to create functions that allow sampling from probability distributions defined by a set of training images.
- Using a Metropolis-adjusted Langevin algorithm allows stochastic inversion with deep generative networks as a prior on spatial property distributions.
- Future work will expand the presented methodology to ill-posed inverse problems for flow and transport such as history matching of hydrocarbon reservoir production data.

References

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